Unit No 3

Ex 3.1 Ques. 1

i) 5700
$= 5.7 \times 10^3$

ii) 49,800,000
$= 498 \times 10^7$

iii) 96,000,000
$= 9.6 \times 10^7$

iv 416.9
$= 4.169 \times 10^2$

v) 83000
$= 8.3 \times 10^4$

vi) 0.00643
$= 6.43 \times 10^{-3}$

vii) 0.0074
$= 7.4 \times 10^{-3}$

viii) 60,000,000
$= 6 \times 10^7$

ix) 0.000000003 95
$= 395 \times 10^{-9}$

x) $\frac{275,000}{0.0025}$
$= \frac{2.75 \times 10^5}{2.5 \times 10^{-3}}$

Ex 3.1 Ques. 2

i) $6 \times 10^{-4} = 0.006$

ii) $5.06 \times 10^{10} = 506\,00000000$

iii) 9.018×10^{-6} 0.000009018

iv) $7.865 \times 10^8 = 786500000$

Ex 3.2

Q1) i) 232.92

log 232.92

log 232.9

Ch = 2

Mentissa = 3672

log 232.9' = 2.3672

2.3299×10^2 → Ch = 2

3655
+ 17
3672

ii) 29.36^2

log 29.326

log 29.33

Ch = 1

mentissa = .4673

log 29.33 = 1.4673

$29.33 = 2.933 \times 10^1$ → Ch

4669
+ 4
4673

iii) 0.00032

log (0.00032)

ch = $\bar{4}$

Mentissa = .5051

log 0.00032 = $\bar{4}$.5051

iv) 0.3206

log 0.3206

ch = $\bar{1}$

Mantissa = .5059

log 0.3206 = $\bar{1}$.5059

5059
+ 8
.5059

Q2) If log 31.09 = 1.4926, find values of the following without using table

i) log 3.109

ch = 0

Man = .4926

log 3.109 = 0.4926

ii) log 310.9

ch = 2

Man = .4926

log 310.9 = 2.4926

iii) log 0.003109

ch = $\bar{3}$

Man = .4926

log 0.003109 = $\bar{3}$.4926

iii) log 0.3109

ch = $\bar{1}$

Man = .4926

log 0.3109 = $\bar{1}$.4926

Ques. 3

i) 3.5621

Let numbers = x

$\log x = 3.5621$

Taking anti log on both sides

$\text{Anti log}(\log x) = \text{Anti log}(3.5621)$

$x = 3649.0$

ii) $\bar{1}.7427$

$\log x = \bar{1}.7427$

$\log x = \bar{1}.7427$

$x = 0.5530$

Q 4 What replacement ....?

i) $\log_3 81 = L$

$3^L = 81$

$3^L = 3^4$

$\boxed{L = 4}$

ii) $\log 9 = x$

$81^x = 9$

$(9^x = 3^2$

ii) $\log a^6 = 0.5$

$\log a^6 = 5/10$ ' 2

$\log a^6 = 1/2$

Write in exponential form

$a^{1/2} = 6$

$\sqrt{a} = 6$

Taking Square on both sides

$(\sqrt{a})^2 = (6)^2$

$a = 36$

iii) $\log 5^{n} = 2$

Write in exponential form

$5^{2} = n$

$\boxed{25 = n}$

iv) $10^{p} = 40$

~~Taking log on both sides~~ Write in logarithmic form

$\log_{10} 40 = p$

$1.6021 = p$

$p = 1.6021$

5 Evaluate $\log_2 \frac{1}{128}$

Let

$$\log_2 \frac{1}{128} = x$$

Write in exponential form.

$$2^x = \frac{1}{128}$$

$$2^x = \frac{1}{2^7}$$

$$2^x = 2^{-7}$$

$$\therefore x = -7 \text{ Ans}$$

Imp

ii log 512 to the base $2\sqrt{2}$

$\log_{2\sqrt{2}} 512$

Let $\log_{2\sqrt{2}} 512 = x$

Write in exponential form.

$(2\sqrt{2})^x = 512$

$(2 \cdot 2^{\frac{1}{2}})^x = 512$

$2^{1+\frac{1}{2}} = \frac{2+1}{2} = \frac{3}{2} = 2^9$

$2^{\frac{3x}{2}} = 2^9$

$\frac{3x}{2} = 9$

$3x = 9 \times 2$

$3x = 18$

$x = \frac{18}{3}$

$\boxed{x = 6}$

Ques. 6) i) $\log_2 x = 5$ $\quad 2^5 = x$

Write in exponential form $\quad 32 = x$

$2^5 = x$

$32 = x$

ii) $\log_{81} 9 = x$ $\quad \log_{81} 9 = x$

Write in exponential form

$(81)^x = 9$ $\quad 81^x = 9$

$(9^2)^x = 9^1$ $\quad (9^2)^x = 9$

$9^{2x} = 9^1$ $\quad 9^{2x} = 9^1$

$2x = 1$ $\quad 2x = 1$

$x = 1/2$ $\quad x = \frac{1}{2}$

iii) $\log_{64} 8 = x/2$

Write in exponential form

$64^{x/2} = 8$

$(8^2)^{x/2} = 8$ $\quad 8^{2 \times x/2} = 8$

$8^x = 8^1$

$x = 1$ Ans

iv $\log_x 64 = 2$

Write in exponential form

$x^2 = 64$

$x^2 = (8)^2$

$x = 8$ Ans

v) $\log_3 x = 4$ Write in exponential form

$3^4 = x$

$81 = x$

$\boxed{x = 81}$

Review ex 3 Ques. 3

i) $\log_3 x = 5$

$3^5 = x$ Write in exponential form

$243 = x$

$x = 243$

ii) $\log_4 256 = x$

Write in exponential form

$4^x = 256$

$4^x = 4^4$

$x = 4$

iii) $\log_{625} 5^{-} = 1/4x$

Write in exponential form

$(625)^{1/4x} = 5$

$(5^4)^{1/4x} = 5$

$5^{4 \times 1/4 x} = 5$

$5^x = 5^1$

$\boxed{x = 1}$

iv) $\log_{64} x = -2/3$

Write in exponential form

$(64)^{-2/3} = x$

$(2^6)^{-2/3} = x$

$2^{6 \times (-2/3)} = x$

$2^{-4} = x$

$\frac{1}{2^4} = x$

$\boxed{1/16 = x}$

Ex 2 Laws Proof

$\log_a(mn) = \log a^m + \log a^n$

$\log a^m + \log a^n$

let

$\log a^m = x$, $\log a^n = y$

Write in exponential form

$a^x = m \rightarrow (i)$ $\quad a^y = n \rightarrow (ii)$

Multiplying eq-i and eq-ii

$a^x \cdot a^y = mn$

$a^{x+y} = mn$

Write in logarithmic form

$\log_a (mn) = x + y$

$\log a^{mn} = \log a^m + \log a^n$

ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

Let

$\log_a m = x$, $\log_a n = y$

Write in exponential form

$a^x = m \rightarrow$ (i) $\quad a^y = n \rightarrow$ ii

Dividing eq - i and - eq - ii

$$\frac{a^x}{a^y} = \frac{m}{n}$$

$$a^{x-y} = \frac{m}{n}$$

Write in logarithm form

$$\log_a \frac{m}{n} = x - y$$

$$\boxed{\log_a \frac{m}{n} = \log_a m - \log_a n}$$

3rd law

$\log_a (m)^n = n \log_a m$

Let

$\log_a m^n = x$, $\log_a m = y$

write in exponential form

$a^x = m^n$

$a^x = (a^y)^n$

$a^x = a^{ny}$

$x = ny$

$\boxed{\log_a m^n = n \log_a m}$

Ex 3.3 Ques. 1

Write the following into sum or difference

i) $\log (A \times B)$

$= \log A + \log B$ Ans.

ii) $\log \frac{16.2}{30.5}$

$= \log 15.2 - \log 30.5$ Ans.

iii) $\log \frac{21 \times 5}{8}$

$= \log (21 \times 5) - \log 8$

$= \log 21 + \log 5 - \log 8$ Ans.

iv) $\log 3\sqrt{\frac{7}{15}}$

$= \log\left(\frac{7}{15}\right)^{1/3}$

$= \frac{1}{3}\log\left(\frac{7}{15}\right)$

$= \frac{1}{3}[\log 7 - \log 15]$ — Bracket must

v) $\log \frac{(22)^{1/3}}{5^3}$

$= \log(22)^{1/3} - \log 5^3$

$= \frac{1}{3}\log 22 - 3\log 5$

Ques. 2 Imp

$$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$$

$$= \log x - \log x^2 + \log(x+1)^3 - \log x^2 - 1$$

$$= \log \frac{x(x+1)^3}{x^2(x^2-1)}$$

$$= \log \frac{(x+1)^3}{x^{2-1}[(x)^2-(1)^2]}$$

$$= \log \frac{(x+1)^3}{x(x+1)(x-1)}$$

$$= \log \frac{(x+1)^{3-1}}{x(x-1)}$$

$$= \log \frac{(x+1)^2}{x(x-1)}$$ Ans

Ex 3.3 Ques. 3

i) $\log 21 + \log 5$

$= \log(21 \times 5)$

ii) $\log 25 - 2\log 3$

$= \log 25 - \log 3^2$

$= \log\left(\frac{25}{3^2}\right)$

iii) $2\log x - 3\log y$

$= \log x^2 - \log y^3$

$= \log\left(\frac{x^2}{y^3}\right)$

iv) $\log 5 + \log 6 - \log 2$

$= \log\frac{(5 \times 6)}{2}$

Ques 4

i $\log_3 2 \times \log_2 81$

$= \frac{\log 2}{\log 3} \times \frac{\log 81}{\log 2}$

$= \frac{\log 81}{\log 3}$

$= \frac{\log 3^4}{\log 3}$

$= 4 \frac{\log 3}{\log 3}$ $\boxed{= 4}$ Ans

ii $\log_5 3 \times \log_3 25$

$= \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 3}$

$= \frac{\log 25}{\log 5}$

$= \frac{\log 5^2}{\log 5}$

$= \frac{2 \log 5}{\log 5}$

2 Ans

Imp Ex 3.3 Ques 5

If $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6990$ then find the value of following.

i $\log 32$

$= \log 2^5$

$= 5 \log 2$

$= 5(0.3010)$

$= 1.5050$

2	32
2	16
2	8
2	4
2	2
	1

ii $\log 24$

$= \log(2^3 \times 3)$

$= \log 2^3 + \log 3$

$= 3 \log 2 + \log 3$

$= 3(0.3010) + (0.4771)$

$= 0.9030 + 0.4771$

$= 1.3801$

2	24
2	12
2	6
3	3
	1

Imp

iii $\log \sqrt{\frac{3}{3}}$

$= \log \sqrt{\frac{10}{3}}$

$= \log \left(\frac{10}{3}\right)^{1/2}$

$= \frac{1}{2} \log \left(\frac{10}{3}\right)$

$= \frac{1}{2}[\log 10 - \log 3]$

$= \frac{1}{2}[\log(2 \times 5) - \log 3]$

$= \frac{1}{2}[\log 2 + \log 5 - \log 3]$

$= \frac{1}{2}[0.3010 + 0.6990 - 0.4771]$

$= \frac{1}{2}[0.5229]$

Round off

$= 0.26145 = 0.2615$

2	8
2	4
2	2
	1

iv $\log \frac{8}{3}$

$= \log 8 - \log 3$

$= \log 2^3 - \log 3$

$= 3\log 2 - \log 3$

$= 3(0.3010) - 0.4771$

$= 0.903 - 0.4771$

$= 0.4259$

v $\log 30$

2	30
3	15
5	5
	1

$= \log(2 \times 3 \times 5)$

$= \log 2 + \log 3 + \log 5$

$= 0.3010 + 0.4771 + 0.6990$

$= 0.7781 + 0.6990$

$= 1.4771$ Ans

Exercise 3.4 Ques. 1

i) 0.8176×13.64

Let

$x = 0.8176 \times 13.64$

Taking log on both sides

$\log x = \log (0.8176 \times 13.64)$

$= \log 0.8176 + \log 13.64$

$= \bar{1}.9125 + 1.1348$

$= -1 + 0.9125 + 1 + 0.1348$

$\log x = 1.0473$

Taking antilog on both sides

$\text{Antilog}(\log x) = \text{Anti log}(1.0473)$

$x =$ Ans

11.15

جب Point کے بعد ایک digit ہو تو اس 1 add کر دو 1 کے اندر

ii) $(789.5)^{1/8}$

Let $x = (789.5)^{1/8}$ add 1

Taking log on both sides

$\log x = \log (789.5)^{1/8}$

$= \frac{1}{8} \log (789.5)$

$= \frac{1}{8} (2.8974) = \frac{2.8974}{3}$

$\log x = 0.362R$

Taking anti log on both sides

$\text{Anti log}(\log x) = \text{Anti log}(0.3622)$

$x = 2.302$

iii) $\dfrac{0.678 \times 9.01}{0.0234}$

Let $x = \dfrac{0.678 \times 9.01}{0.0234}$

Taking log on both sides

$\log x = \log\left(\dfrac{0.678 \times 9.01}{0.0234}\right)$

$\log(0.678 \times 9.01) - \log 0.0234$

$= \log 0.678 + \log 9.01 - \log 0.0234$

$= \bar{1}.8312 + 0.9547 - \bar{2}.3692$

$= -1 + 0.8312 + 0.9547 - (-2 + 0.3692)$

$= -1 + 0.8312 + 0.957 + 2 - 0.3692$

$= -1 + 2 + 0.8312 + 0.957 - 0.362$

$= 1 + 1.4167$

$\log x = 2.4167$

Taking anti log on both sides

$\text{Anti log}(\log x) = \text{Anti log}(2.4167)$

$x = 261.0$

decimal lagane k liye ch main 1 add kr dete (them) $2+1=3$ is liye $261 = 261.0$

iv) $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Let $x = \sqrt[5]{2.709} \times \sqrt[7]{1.239}$

$= (2.709)^{1/5} \times (1.239)^{1/7}$

Taking log on both sides

$\log x = \log [(2.709)^{1/5} \times (1.239)^{1/7}]$

$= \log (2.709)^{1/5} + \log (1.239)^{1/7}$

$= \frac{1}{5} \log 2.709 + \frac{1}{7} \log (1.239)$

$= \frac{1}{5}(0.4328) + \frac{1}{7}(0.0931)$

$= \frac{0.4328}{5} + \frac{0.0931}{7}$

$= 0.0865 + 0.0133$

$\log x = 0.0998$

Taking anti log on both sides

Anti log $(\log x)$ = Anti log (0.0998)

$x = 1.258$

Imp

v) $\frac{(1.23)(0.6975)}{(0.0075)(1278)}$

Let $x = \frac{(1.23)(0.6975)}{(0.0075)(1278)}$

Taking log on both sides

$\log x = \log \frac{(1.23)(0.6975)}{(0.0075)(1278)}$

$= \log[(1.23)(0.6975)] - \log[(0.0075)(1278)]$

$= \log 1.23 + \log 0.6975 - [\log 0.0075 + \log 1278]$

$= \log 1.23 + \log 0.6975 - \log 0.0075 - \log 1278$

$= 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$

$= 0.0899 - 1 + 0.8435 - (-3 + 0.8751) - 3 - 0.1065$

$= 0.0899 - 1 + 0.8435 + 3 - 0.8751 - 3 - 0.1065$

$= -1 + 0.0899 + 0.8435 - 0.8751 - 0.1065$

$= -1 - 0.0482$

$= -1 - 1 + 1 - 0.0482$

$= -2 + 0.9518$

$\log x = \bar{2}.9518$

Taking anti log on both sides

Anti log (log x) = Anti log ($\bar{2}$.9518)

$x = 0.08950$

vi) $\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$

Let $x = \left(\frac{0.7214 \times 20.37}{60.8}\right)^{1/3}$

Taking log on B.S

$$\log x = \log\left(\frac{0.7214 \times 20.37}{60.8}\right)^{1/3}$$

$$= \frac{1}{3}\log\left(\frac{0.7214 \times 20.37}{60.8}\right)$$

$$= \frac{1}{3}\left[\log(0.7214 \times 20.37) - \log 60.8\right]$$

$$= \frac{1}{3}\left[\log 0.7214 + \log 20.37 - \log 60.8\right]$$

$$= \frac{1}{3}\left[\bar{1}.8581 + 1.3090 - 1.7839\right]$$

جب اول Minus (-) ہو تو ($\bar{1}$) تو پھر 0 کے ساتھ Plus 3 ہوتا ہے (1+0.30)

اور اگر اس کے ساتھ Minus ہو تو 0 بھی Minus ہوتا ہے

$$= \frac{1}{3}\left[-1 + 0.8581 + 1 + 0.3090 - 1 - 0.7839\right]$$

$$= \frac{1}{3}\left[-1 + 0.8581 + 0.3090 - 0.7839\right]$$

$$= \frac{1}{3}\left[-1 + 0.3832\right]$$

$$= \frac{1}{3}\left[-0.6168\right]$$

$$= -\frac{0.6168}{3}$$

$= -0.2056$

$= -1 + 1 - 0.2056$

$= -1 + 0.7944$

$\log x = \bar{1}.7944$

Taking anti log on B.S

$\text{Anti log}(\log x) = \text{Anti log}(\bar{1}.7944)$

$x = 0.6229$

6229

vii) $\dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

viii

Let $x = \dfrac{83 \times (92)^{1/3}}{127 \times (246)^{1/5}}$

Taking

$\log x = \log \dfrac{83 \times (92)^{1/3}}{127 \times (246)^{1/5}}$

$= \log [83 \times (92)^{1/3}] - \log [127 \times (246)^{1/5}]$

$= \log 83 + \dfrac{1}{3} \log 92 - \log 127 - \dfrac{1}{5} \log 246$

$= 1.9191 + \dfrac{1.9638}{3} - 2.1038 - 0.4782$

$= 1 + 0.9191 + 0.6546 - 2 - 0.1038 - 0.4782$

$= -1 + 0.9917$

$\log x = \bar{1}.9917$

Anti log $\bar{1}$ = Anti log (T.9997)

x = 0.9817

5. If $V = \frac{1}{3}\pi r^2 h$, ...

REVIEW EXERCISE 3

1. Multiple Choice Questions. Choose the correct answer.

(i) If $a^x = n$, then .....

(a) $a = \log_x n$ (b) $x = \log_n a$ (c) $x = \log_a n$ (d) $a = \log_n x$

(ii) The relation $y = \log_z x$ implies .....

(a) $x^y = z$ (b) $z^y = x$ (c) $x^z = y$ (d) $y^z = x$

(iii) The logarithm of unity to any base is .....

(a) 1 (b) 10 (c) e (d) 0

(iv) The logarithm of any number to itself as base is .....

(a) 1 (b) 0 (c) -1 (d) 10

(v) $\log e$ = ..., where $e \approx 2.718$

(a) 0 (b) 0.4343 (c) ∞ (d) 1

(vi) The value of $\log\left(\frac{p}{q}\right)$ is .....

(a) $\log p - \log q$ (b) $\frac{\log p}{\log q}$

(c) $\log p + \log q$ (d) $\log q - \log p$

(vii) $\log p - \log q$ is same as .....

(a) $\log\left(\frac{q}{p}\right)$ (b) $\log (p - q)$ (c) $\frac{\log p}{\log q}$ (d) $\log\left(\frac{p}{q}\right)$

(viii) $\log (m^n)$ can be written as .....

(a) $(\log m)^n$ (b) $m \log n$ (c) $n \log m$ (d) $\log (mn)$

(ix) $\log_b a \times \log_c b$ can be written as .....

(a) $\log_a c$ (b) $\log_c a$ (c) $\log_a b$ (d) $\log_b c$

(x) $\log_y x$ will be equal to .....

(a) $\frac{\log_z x}{\log_y z}$ (b) $\frac{\log_x z}{\log_y z}$ (c) $\frac{\log_z x}{\log_z y}$ (d) $\frac{\log_z y}{\log_z x}$