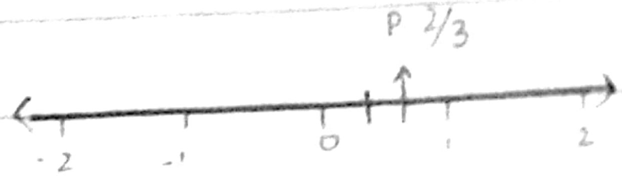
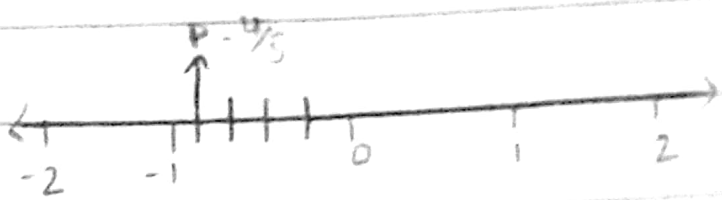
CH # 2 Exercise 2.1 Ques. 4

Represent _ _ _ _ _ _ _ _ _ line.

1) $\frac{2}{3}$

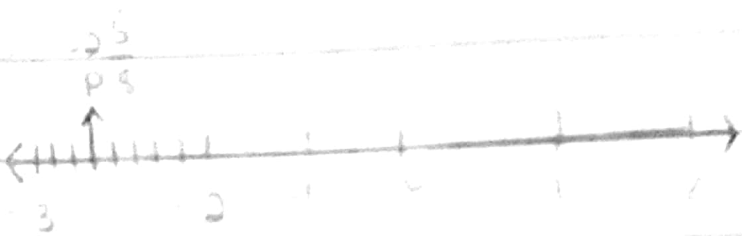


2) $-\frac{4}{5}$



3) $1\frac{3}{4}$

4) $-2\frac{5}{8}$



5) $2\frac{3}{4}$

Ex 2.1 Ques. 5

Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$.

Solution:

$$= \frac{\frac{3}{4} + \frac{5}{9}}{2}$$

$$= \frac{1}{2}\left(\frac{3}{4} + \frac{5}{9}\right)$$

$$= \frac{1}{2}\left(\frac{27+20}{36}\right)$$

$$= \frac{1}{2}\left(\frac{47}{36}\right)$$

$$= \frac{41}{72} \text{ Ans}$$

Ques. 6 Express the recurring decimals as the rational number $\frac{p}{q}$ where

) $0.\overline{5}$

Solution:

Let

$$x = 0.\overline{5}$$

$$x = 0.5555\ldots\ldots \longrightarrow ①$$

multiplying eq ① by 10

$10x = 5.5555....$ ②

Subtract eq ① from eq ②

$10x - x = (5.5555...) - (0.5555....)$

$9x = 5$

$\boxed{x} = \frac{5}{9}$ Ans

ii) $0.\overline{13}$

Solution:

Let $x = 0.\overline{13}$

$x = 0.131313.....$ → ①

Multiplying eq ① by 100

$100x = 13.131313.....$ → ②

Subtract eq ① from eq ②

$100x - x = (13.131313.....) - (0.131313.....)$

$99x = 13$

$\boxed{x} = \frac{13}{9}$ Ans

iii) $0.\overline{67}$

Solution:-

Let $x = 0.\overline{67}$

$x = 0.676767\ldots \rightarrow (1)$

Multiplying eq (1) by 100

$100x = 67.676767\ldots \rightarrow (2)$

Subtract eq (1) from eq (2)

$100x - x = (67.676767\ldots) - (0.676767\ldots)$

$99x = 67$

$\boxed{x} = \frac{67}{99}$ **Ans**

Exercise 2.3 Ques. 3

i) $\sqrt[3]{-125}$

$= \sqrt[3]{-5^3}$

$= -5$ Ans

ii) $\sqrt[4]{32}$

$= \sqrt[4]{2^4 \times 2}$

$= \sqrt[4]{2^4} \times \sqrt[4]{2}$

$= 2 \times \sqrt[4]{2}$ Ans

iii) $\sqrt[5]{\frac{3}{32}}$

$= \frac{\sqrt[5]{3}}{\sqrt[5]{32}}$

$= \frac{\sqrt[5]{3}}{\sqrt[5]{2^5}}$

$= \frac{\sqrt[5]{3}}{2}$ Ans

iv $\sqrt[3]{\frac{8}{27}}$

$= \sqrt[3]{\frac{-2^3}{3^3}}$

$= \sqrt[3]{\left(-\frac{2}{3}\right)^3} = -\frac{2}{3}$

iv) Imp) $\dfrac{(81)^n \cdot 3^5 - 3^{4n-1}(243)}{(9^{2n})\,(3^3)}$

ہر کو Base میں سے آرہی۔
زیادہ 3 ہے اس لیے
3 میں زیادہ Convert کیا۔

$$= \frac{(3^4)^n \cdot 3^5 - 3^{4n-1} \cdot 3^5}{(3^2)^{2n} \quad (3^3)}$$

$$= \frac{3^{4n} \cdot 3^5 - 3^{4n-1} \cdot 3^5}{3^{4n} \times 3^3}$$

$$= \frac{3^{4n+5} - 3^{4n-1+5}}{3^{4n+3}}$$

$$= \frac{3^{4n+5} - 3^{4n+4}}{3^{4n+3}}$$

$$= \frac{3^{4n+5}}{3^{4n+3}} - \frac{3^{4n+4}}{3^{4n+3}}$$

$$= 3^{\cancel{4n}+5-\cancel{4n}-3} - 3^{\cancel{4n}+4-\cancel{4n}-3}$$

$$= 3^2 - 3^1$$

$$= 9 - 3$$

$$= 6 \text{ Ans}$$

Exercise 2.4 Ques. 2

Show that:

$$\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{c^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = 1$$

Solution:

L.H.S again

$= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a}$, $\because \frac{a^m}{a^n} = a^{m-n}$

$= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$, $\because (a^m)^n = a^{mn}$

$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$, $\because (a+b)(a-b) = a^2-b^2$

$= x^{\cancel{a^2}-\cancel{b^2}+\cancel{b^2}-\cancel{c^2}+\cancel{c^2}-\cancel{a^2}}$, $a^m.a^n = a^{m+n}$

$= x^0 = 1 = R.H.S$ $\therefore a^0 = 1$

Ques.3 Simplify Imp

i) $\frac{2^{1/3} \times (27)^{1/3} \times (60)^{1/2}}{(180)^{1/2} \times (4)^{-1/3} \times (9)^{1/4}}$

$= \frac{2^{1/3} \times (3^3)^{1/3} \times (2^2\times3\times5)^{1/2}}{(2^2\times3^2\times5)^{1/2} \times (2^2)^{-1/3} \times (3^2)^{1/4}}$

$\frac{2^{1/3} \times 3^{\cancel{3}\times\frac{1}{\cancel{3}}} \times 2^{\cancel{2}\times\frac{1}{\cancel{2}}} \times 3^{1/2} \times 5^{1/2}}{2^{\cancel{2}\times\frac{1}{\cancel{2}}} \times 3^{\cancel{2}\times\frac{1}{\cancel{2}}} \times 5^{\frac{1}{2}} \times 2^{-2/3} \times 3^{\cancel{2}\times\frac{1}{\cancel{4}2}}}$

$\frac{2^{1/3} \times \cancel{3} \times \cancel{2} \times \cancel{3^{1/2}} \times \cancel{5^{1/2}}}{\cancel{2} \times \cancel{3} \times \cancel{5^{1/2}} \times 2^{-\frac{2}{3}} \times \cancel{3^{1/2}}}$

$= \frac{2^{1/3}}{2^{-2/3}}$

$= 2^{\frac{1}{3}+\frac{2}{3}}$

$= 2^{\frac{1+2}{3}}$

$= 2^{\cancel{3}/\cancel{3}}$

$= 2^1$

$= 2$ Ans

ii $\sqrt{\frac{(216)^{2/3} \times (25)^{1/2}}{(.04)^{-1/2}}}$

Solution:-

$= \sqrt{(216)^{2/3} \times (25)^{\frac{1}{2}} \times (.04)^{+1/2}}$

$= \sqrt{(2^3 \times 3^3)^{2/3} \times (5^2)^{\frac{1}{2}} \times \left(\frac{\cancel{4}\,1}{\cancel{100}\,25}\right)^{1/2}}$

$= \sqrt{2^{\cancel{3} \times 2/\cancel{3}} \times 3^{\cancel{3} \times \frac{2}{\cancel{3}}} \times 5^{\cancel{2} \times \frac{1}{\cancel{2}}} \times \left(\frac{1}{25}\right)^{\frac{1}{2}}}$

$= \sqrt{2^2 \times 3^2 \times 5 \times \left(\frac{1}{5^2}\right)^{\frac{1}{2}}}$

$= \sqrt{2^2 \times 3^2 \times 5 \times \frac{1}{5^{\cancel{2} \times \frac{1}{\cancel{2}}}}}$

$= \sqrt{2^2 \times 3^2 \times \cancel{5} \times \frac{1}{\cancel{5}}}$

$= \sqrt{2^2 \times 3^2}$

$= \sqrt{2^{\cancel{2}}} \times \sqrt{3^{\cancel{2}}}$

$= 2 \times 3$

$= 6$

iii $5^{2^3} \div (5^2)^3$

$= 5^8 \div 5^6 \quad \because (a^m)^n = a^{mn}$

$= \frac{5^8}{5^6}$

$= 5^{8-6} = 5^2 \quad \because \frac{a^m}{a^n} = a^{m-n}$

$= 25$ Ans

iv $(x^3)^2 \div x^{3^2}$

$= x^6 \div x^9 \quad \because (a^m)^n = a^{mn}$

$= \frac{x^6}{x^9}$

$= \frac{1}{x^{9-6}} \quad \frac{a^{-m}}{} = \frac{1}{a^m}$

$= \frac{1}{x^3}$

Review ex Ques. 4 Simplify

$$\sqrt{\frac{(216)^{2/3} \times (25)^{1/2}}{(0.04)^{-3/2}}}$$

Solution:

$$= \sqrt{(216)^{2/3} \times (25)^{1/2} \times (0.04)^{+3/2}}$$

$$= \sqrt{(2^3 \times 3^3)^{2/3} \times (5^2)^{\frac{1}{2}} \times \left(\frac{\cancel{4}\,1}{\cancel{100}\,25}\right)^{3/2}}$$

$$= \sqrt{2^{\cancel{3} \times \frac{2}{\cancel{3}}} \times 3^{\cancel{3} \times \frac{2}{\cancel{3}}} \times 5^{\cancel{2} \times \frac{1}{\cancel{2}}} \times \frac{1}{25}^{3/2}}$$

$$= \sqrt{2^2 \times 3^2 \times 5 \times \left(\frac{1}{5^2}\right)^{\frac{3}{2}}}$$

$$= \sqrt{2^2 \times 3^2 \times 5 \times \frac{1}{5^{\cancel{2} \times \frac{3}{\cancel{2}}}}}$$

$$= \sqrt{2^2 \times 3^2 \times \cancel{5} \times \frac{1}{5^{3-1}}}$$

$= \sqrt{2^2 \times 3^2 \times 5^2} \qquad = \sqrt{\dfrac{2^2 \times 3^2}{5^2}}$

$= \sqrt{2^2} \times \sqrt{3^2} \times \sqrt{5^2}$

$= \dfrac{2 \times \sqrt{3^2}}{\sqrt{5^2}}$

$= \dfrac{2 \times 3}{5}$

$= \dfrac{6}{5}$ Ans

Review ex 2 Ques 6 Simplify Imp

$$\left(\frac{a^{2l}}{a^{l+m}}\right)\left(\frac{a^{2m}}{a^{m+n}}\right)\left(\frac{a^{2n}}{a^{n+l}}\right)$$

Solution:

$= \left(a^{2l-l-m}\right)\left(a^{2m-m-n}\right)\left(a^{2n-n-l}\right) \qquad \because \dfrac{a^m}{a^n} = a^{m-n}$

$= \left(a^{l-m}\right)\left(a^{m-n}\right)\left(a^{n-l}\right)$

$= a^{\cancel{l}-\cancel{m}+\cancel{m}-\cancel{n}+\cancel{n}-\cancel{l}} \qquad \because a^m \cdot a^n = a^{m+n}$

$= a^0$

$= \underline{1} \qquad \because a^0 = 1$

Exercise 2.5 Ques 1

i) i^7

$= i.(i)^6$

$= i.(i^2)^3$

$= i(-1)^3$

$= i(-1)$

$= -i$

ii i^{50}

$= (i^2)^{25}$

$= (-1)^{25}$

$= -1$

iii) i^{12}

$= (i^2)^6$

$= (-1)^6$

$= 1$ Ans

iv $(-i)^8$

$= (-1 \cdot i)^8$

$= (-1)^8 . (i)^8$

$= 1 . (i^2)^4$

$= (-1)^4$

$= 1$

v) $(-i)^5$

$= (-1 . i)^5$

$= (-1)^5 . (i)^5$

$= -1 . i . i^4$

$= -i \cdot (i^2)^2$

$= -i (-1)^2$

$= -i(1)$

$= -i$

vi) i^{27}

$= i (i)^{26}$

$= i . (i^2)^{13}$

$= i . (-1)^{13}$

$= -i$

Imp

Exercise 2.5 Ques. 4

Find the value of x and y if $x + iy + 1 = 4 - 3i$

$x = ?$

$y = ?$

$x + iy + 1 = 4 - 3i$

$x + iy = 4 - 3i - 1$

$x + iy = 3 - 3i$

Comparing real and imaginary parts

$x = 3 \qquad y = -3$

Ex 2.6 Ques. 2

i) $(2+3i) + (7-2i)$

$= 2 + 3i + 7 - 2i$

$= 2 + 7 + 3i - 2i$

$= 9 + i$ Ans

ii $2(5+4i) - 3(7+4i)$

$= 10 + 8i - 21 - 12i$

$= 10 - 21 + 8i - 12i$

$= -11 - 4i$

iii $-(-3+5i) - (4+9i)$

$= 3 - 5i - 4 - 9i$

$= 3 - 4 - 5i - 9i$

$= -1 - 14i$

iv Imp $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

$= 2(-1) + 6i(i^2) + 3 \cdot (i^2)^8 - 6i(i^2)^9 + 4i$

$= -2 + 6i(-1) + 3(-1)^8 - 6i(i^2)^9 + 4i(i^2)^{12}$

$= -2 - 6i + 3(-1) - 6i(-1)^9 + 4i(-1)^{12}$

$= -2 - 6i + 3 - 6i(-1) + 4i(1)$

$= -2 - \cancel{6i} + 3 + \cancel{6i} + 4i$

$= 1 + 4i$

Ex 2.6 Ques. 3

Simplify and write your answer in the form of $a+bi$

i) $(-7+3i)(-3+2i)$

Solution:

$= -7(-3+2i)+3i(-3+2i)$

$= 21-14i-9i+6i^2$

$= 21-23i+6(-1)$

$= 21-23i-6$

$= 15-23i$

ii $2(5+4i)-3(7+4i)$) x

$= 10+8i-21-12i$

ii) ~~$(\sqrt{2-\sqrt{4}})(3-\sqrt{-4})$~~ $(2-\sqrt{-4})(3-\sqrt{-4})$

$= (\sqrt{2-\sqrt{4(-1)}})(3-\sqrt{4(-1)})$

$= (2-\sqrt{4}\cdot\sqrt{-1})(3-\sqrt{4}\sqrt{(-1)})$

$= (2-2i)(3-2i)$ $\quad \therefore \sqrt{-1} = i$

$= 6-4i-6i+4i^2$

$= 6-10i+4(-1)$

$= 6-10i-4$

$= 2-10i$ Ans

iii $(\sqrt{5} - 3i)^2$ $\therefore (a-b)^2 = a^2 + b^2 - 2ab$

$= (\sqrt{5})^2 + (3i)^2 - 2\sqrt{5}(3i)$

$= 5 + 9i^2 - 6\sqrt{5}i$

$= 5 + 9(-1) - 6\sqrt{5}i$

$= 5 - 9 - 6\sqrt{5}i$

$= -4 - 6\sqrt{5}i$

iv $(2-3i)\overline{(3-2i)}$

$= (2-3i)(3+2i)$

$= 6 + 4i - 9i - 6i^2$

$= 6 - 5i - 6(-1)$

$= 6 - 5i + 6$

$= 12 - 5i$

Ex No 2.6 Ques. 4

i) $\dfrac{-2}{1+i}$

$= \dfrac{-2}{1+i} \times \dfrac{1-i}{1-i}$

$= \dfrac{-2(1-i)}{(1+i)(1-i)}$

$= \dfrac{-2(1-i)}{(1)^2 - i^2}$

$= \dfrac{-2(1-i)}{1-(-1)}$

$= \dfrac{-2(1-i)}{1+1}$

$\dfrac{-\cancel{2}(1-i)}{\cancel{2}}$

$-1+i$

ii) $\dfrac{2+3i}{4-i}$

$= \dfrac{2+3i}{4-i} \times \dfrac{4+i}{4+i}$

$= \dfrac{(2+3i)(4+i)}{(4-i)(4+i)}$

$= \dfrac{8+2i+12i+3i^2}{(4)^2 - i^2}$

$= \dfrac{8+14i+3(-1)}{16-(-1)}$

$= \dfrac{8+14i-3}{17}$

$= \dfrac{8-3+14i}{17}$

$= \dfrac{5+14i}{17}$

$= \dfrac{5}{17} + \dfrac{14}{17}i$

iv $\frac{2-6i}{3+i} - \frac{4+i}{3+i}$

$= \frac{2-6i-(4+i)}{3+i}$

$= \frac{2-6i-4-i}{3+i}$

$= \frac{2-4-6i-i}{3+i}$

$= \frac{-2-7i}{3+i} \times \frac{3-i}{3-i}$

$= \frac{(-2-7i)(3-i)}{(3+i)(3-i)}$

$= \frac{-6+2i-21i+7i^2}{(3)^2-i^2}$

$= \frac{-6-19i+7(-1)}{9-(-1)}$

$= \frac{-6-19i-7}{9+1}$

$= \frac{-6-7-19i}{10}$

$= \frac{-13-19i}{10}$

$= \frac{-13}{10} - \frac{19}{10}i$

v) $\frac{(1+i)^2}{(1-i)^2}$

$= \frac{(1+i)^2}{(1-i)^2}$

$= \frac{(1)^2+(i)^2+2(1)(i)}{(1)^2+(i)^2-2(1)(i)}$

$= \frac{1+i^2+2i}{1+i^2-2i}$

$= \frac{\cancel{1}-\cancel{1}+2i}{\cancel{1}-\cancel{1}-2i}$

$= \frac{\cancel{2i}\ 1}{-\cancel{2i}\ 1}$

$= -1$

$= -1+0i$

vi) $\dfrac{1}{(2+3i)(1-i)}$

$= \dfrac{1}{2-2i+3i-3i^2}$

$= \dfrac{1}{2+i-3(-1)}$

$= \dfrac{1}{2+i+3}$

$= \dfrac{1}{5+i}$

$= \dfrac{1}{5+i} \times \dfrac{5-i}{5-i}$

$= \dfrac{5-i}{(5)^2-(i)^2}$

$= \dfrac{5-i}{25-(-1)}$

$= \dfrac{5-i}{25+1}$

$= \dfrac{5-i}{26}$

$= \dfrac{5}{26} - \dfrac{1}{26}i$

Q# 5 i) $z = -i$

a) $\bar{z} = i$

b) $z+\bar{z} = -i+i = 0$

c) $z-\bar{z} = -i-i = -2i$

d) $z\cdot\bar{z} = (-i)(i)$

$= (-i^2)$

$-(-1) = 1$

ii) $2+i$

a) $\bar{z} = 2-i$

b) $z+\bar{z} = 2+\cancel{i}+2-\cancel{i}$

$= 4$

c) $z-\bar{z} = 2+i-(2-i)$

$= \cancel{2}+i-\cancel{2}+i$

$= 2i$

d) $z\cdot\bar{z} = (2+i)(2-i)$

$= (2)^2 - i^2$

$= 4-(-1)$

$= 4+1 = 5$

iii $z = \frac{1+i}{1-i}$

$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$z = \frac{(1+i)^2}{(1-i)(1+i)}$

$z = \frac{(1+i^2)}{(1)^2-(i)^2}$

$z = \frac{(1)^2+i^2+2(1)(i)}{1-(-1)}$

$z = \frac{\cancel{1}+\cancel{(-1)}+2i}{1+1}$

$z = \frac{\cancel{2}i}{\cancel{2}}$

$z = i$

$\bar{z} = -i$

$z+\bar{z} = i-i = 0$

$z-\bar{z} = i-(-i) = i+i = 2i$

$z \cdot \bar{z} = (i)(-i) = (-i)^2 = -(-1)$

$= 1$

iv) $z = \frac{4-3i}{2+4i}$

$z = \frac{4-3i}{2+4i} \times \frac{2-4i}{2-4i}$

$= \frac{(4-3i)(2-4i)}{(2)^2-(4i)^2}$

$= \frac{8-16i-6i+12i^2}{4-16i^2}$

$= \frac{8-22i+12(-1)}{4-16(-1)}$

$= \frac{8-22i-12}{4+16}$

$= \frac{-4-22i}{20}$

$= \frac{-\cancel{4}^{1}}{\cancel{20}_{5}} - \frac{\cancel{22}i^{11}}{\cancel{20}_{10}}$

$= \frac{-1}{5} - \frac{11i}{10}$

$\bar{z} = \frac{-1}{5} + \frac{10i}{10}$

$z+\bar{z} = -\frac{1}{5} - \cancel{\frac{11}{10}i} - \frac{1}{5} + \cancel{\frac{11}{10}i}$

$= -\frac{1}{5} - \frac{1}{5} = \frac{-1-1}{5}$

$= \frac{-2}{5}$.

$z-\bar{z} = -\frac{1}{5} - \frac{11}{10}i - \left(-\frac{1}{5} + \frac{11}{10}i\right)$

$= \cancel{-\frac{1}{5}} - \frac{11}{10}i + \cancel{\frac{1}{5}} - \frac{11}{10}i$

$= \frac{-11i}{10} - \frac{11i}{10}$

$= \frac{-11i - 11i}{10}$

$= \frac{22i}{10} = \frac{11i}{5}$

$z \cdot \bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right)\left(-\frac{1}{5} + \frac{11}{10}i\right)$

$= \left(\frac{-1}{5}\right)^2 - \left(\frac{11}{10}i\right)^2$

$= \frac{1}{25} - \frac{121i^2}{100}$

$= \frac{1}{25} - \frac{121(-1)}{100}$

$= \frac{1}{25} + \frac{121}{100}$

$= \frac{4+121}{100}$

$= \frac{125}{100} = \frac{5}{4}$

Ques.6 If $z = 2+3i$ and $w = 5-4i$ show that

i) $\overline{Z+W} = \overline{Z} + \overline{W}$

L.H.S $= \overline{Z+W}$

$Z+W = (2+3i)+(5-4i)$

$= 2+3i+5-4i$

$= 2+5+3i-4i$

$Z+W = 7-i$

$\overline{Z+W} = 7+i \longrightarrow (1)$

R.H.S $\overline{Z}+\overline{W}$

$\overline{Z}+\overline{W} = \overline{(2+3i)}+\overline{(5-4i)}$

$\overline{Z}+\overline{W} = (2-3i)+(5+4i)$

$\overline{Z}+\overline{W} = 2-3i+5+4i$

$\overline{Z}+\overline{W} = 2+5-3i+4i$

$\overline{Z}+\overline{W} = 7+i$ Ans $\longrightarrow (2)$

From eq (1) and (2)

$\overline{Z+W} = \overline{Z}+\overline{W}$

Hence Proved.

ii) $\overline{Z-W} = \overline{Z} - \overline{W}$

L.H.S $= \overline{Z-W}$

$Z-W = (2+3i)-(5-4i)$

$= 2+3i-5+4i$

$= 2-5+3i+4i$

$Z-W = -3+7i$

$\overline{Z-W} = -3-7i \longrightarrow (1)$

R.H.S $= \overline{Z} - \overline{W}$

$\overline{Z} - \overline{W} = (\overline{2+3i}) - (\overline{5-4i})$

$\overline{Z} - \overline{W} = (2-3i)-(5+4i)$

$\overline{Z-W} = 2-3i-5-4i$

$\overline{Z} - \overline{W} = 2-5-3i-4i$

$\overline{Z} - \overline{W} = -3-7i \longrightarrow (2)$

From eq 1 and 2

L.H.S = R.H.S Hence Proved

iii $\overline{ZW} = \overline{Z} \cdot \overline{W}$

L.H.S $\overline{ZW}$

$ZW = (2+3i)(5-4i)$

$ZW = 10-8i+15i-12i^2$

$ZW = 10+7i-12(-1)$

$ZW = 10+7i+12$

$ZW = 22+7i$

$\overline{ZW} = 22-7i \longrightarrow (1)$

iii) R.H.S $\bar{z}\,\bar{w}$

$\bar{z}\,\bar{w} = \overline{(2+3i)}\,\overline{(5-4i)}$

$\bar{z}\,\bar{w} = (2-3i)(5+4i)$

$\bar{z}\,\bar{w} = 10+8i-15i-12i^2$

$\bar{z}\,\bar{w} = 10-7i-12(-1)$

$\bar{z}\,\bar{w} = 10-7i+12$

$\bar{z}\,\bar{w} = 22-7i$ Hence Proved

iv) $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$ where $w \neq 0$

L.H.S $\overline{\left(\frac{z}{w}\right)}$

$\left(\frac{z}{w}\right) = \frac{(2+3i)}{(5-4i)} \times \frac{5+4i}{5+4i}$

$\left(\frac{z}{w}\right) = \frac{(2+3i)(5+4i)}{(5-4i)(5+4i)}$

$\left(\frac{z}{w}\right) = \frac{10+8i+15i+12i^2}{5^2-(4i)^2}$

$\left(\frac{z}{w}\right) = \frac{10+23i+12(-1)}{25-16i^2}$

$\left(\frac{z}{w}\right) = \frac{10+23i-12}{25-16(-1)}$

$\left(\frac{z}{w}\right) = \frac{-2+23i}{25+16}$

$\left(\frac{z}{w}\right) = \frac{-2+23i}{41} = \frac{-2}{41} + \frac{23i}{41}$

$$\left(\overline{\frac{z}{w}}\right) = \frac{-2}{41} - \frac{23}{41}i \longrightarrow (1)$$

R.H.S $\dfrac{\overline{z}}{\overline{w}}$

$$\frac{\overline{z}}{\overline{w}} = \frac{(\overline{2+3i})}{(\overline{5-4i})}$$

$$\frac{\overline{z}}{\overline{w}} = \frac{(2-3i)}{(5+4i)} \times \frac{(5-4i)}{(5-4i)}$$

$$\frac{\overline{z}}{\overline{w}} = \frac{(2-3i)(5-4i)}{(5+4i)(5-4i)}$$

$$\frac{\overline{z}}{\overline{w}} = \frac{2(5-4i)-3i(5-4i)}{5^2-(4i)^2}$$

$$\frac{\overline{z}}{\overline{w}} = \frac{10-8i-15i+12i^2}{25-16i^2}$$

$$\frac{\overline{z}}{\overline{w}} = \frac{10-23i+12(-1)}{25-16(-1)}$$

$$\frac{\overline{z}}{\overline{w}} = \frac{10-23i-12}{25+16}$$

$$\frac{\overline{z}}{\overline{w}} = \frac{-2-23i}{41}$$

$$\frac{\overline{z}}{\overline{w}} = \frac{-2}{41} - \frac{23}{41}i \longrightarrow (2)$$

From eq (1) and (2)

L.H.S = R.H.S

Hence Proved

v) $\frac{1}{2}(z+\bar{z})$ is real part of z

$\bar{z} = 2-3i$

$\frac{1}{2}(z+\bar{z}) = \frac{1}{2}[(2+3i)+(2-3i)]$

$= \frac{1}{2}(2+\cancel{3i}+2-\cancel{3i})$

$= \frac{1}{\cancel{2}}(\cancel{4})2$

$= 2$

So $\frac{1}{2}(z+\bar{z})$ is the real part of z.

vi) $\frac{1}{2i}(z-\bar{z})$ is the imaginary part.

$\bar{z} = 2-3i$

$\frac{1}{2i}(z-\bar{z}) = \frac{1}{2i}[(2+3i)-(2-3i)]$

$= \frac{1}{2i}[\cancel{2}+3i-\cancel{2}+3i]$ $\cancel{3i+3i}$

$= \frac{1}{\cancel{2i}}(\cancel{6i})^{3}$ $\cancel{3i+3i}\,i$

$\cancel{6i}$

$\frac{1}{2i}(z-\bar{z}) = 3$

Imp

~~(2~~ Ques. 7

i) $(2-3i)(x+yi) = 4+i$

$$(x+yi) = \frac{4+i}{2-3i}$$

$$x+yi = \frac{4+i}{2-3i} \times \frac{2+3i}{2+3i}$$

$$x+yi = \frac{(4+i)(2+3i)}{(2-3i)(2+3i)}$$

$$x+yi = \frac{8+12i+2i+3i^2}{4-9i^2}$$

$$x+yi = \frac{8+14i+3(-1)}{4-9(-1)}$$

$$x+yi = \frac{8+14i-3}{4+9}$$

$$x+yi = \frac{5+14i}{13}$$

$$x+yi = \frac{5}{13} + \frac{14i}{13}$$

Comparing real part ~~and~~ imaginary parts

$$x = \frac{5}{13}, \quad y = \frac{14}{13}$$

ii) $(3-2i)(x+yi) = 2(x-2yi)+2i-1$

$3x+3yi-2xi-2yi^2 = 2x-4yi+2i-1$

$3x+3yi-2xi-2y(-1)-2x+4yi = 2i-1$

$3x+3yi-2xi+2y-2x+4yi = 2i-1$

$3x-2x+3yi+4yi-2xi+2y = -1+2i$

$x+7yi-2xi+2y = -1+2i$

$(x+2y)+(7y-2x)i = -1+2i$

By comparing real and imaginary parts

$x+2y = -1 \rightarrow (1)$

$7y-2x = 2$

$-2x+7y = 2 \rightarrow (2)$

Multiplying eq (1) by 2

$2x+4y = -2 \rightarrow (3)$

Adding eq (2) and (3)

$-\cancel{2x}+7y = 2$

$\cancel{2x}+4y = -2$

$11y = 0$

$11y = 0$

$y = \frac{0}{11}$

$y = 0$

$\boxed{y = 0}$

Put $y=0$ in eq (1)

$x+2(0) = -1$

$x+0 = -1$

$\boxed{x = -1}$

iii) $(3+4i)^2 - 2(x-yi) = x+yi$

$\bullet\ (3+4i)^2 = x+yi+2(x-yi)$

$(3)^2+(4i)^2+2(3)(4i) = x+yi+2x-2yi)$

$9+16i^2+24i = x+2x+yi-2yi$

$9+16(-1)+24i = 3x-yi$

$9-16+24i = 3x-yi$

$-7+24i \quad 3x-yi$

Comparing real and imaginary parts

$-7 = 3x \quad , \quad 24 = -y$

$\frac{-7}{3} = x \quad\quad -24 = y$

$\boxed{y = -24}$

$\boxed{x = \frac{-7}{3}}$

Ans

Review Ex 2

3) i) $\sqrt[4]{81y^{-12}x^{-8}}$

$= \left(\dfrac{81}{y^{12}x^{8}}\right)^{1/4}$

$= \left(\dfrac{3^4}{x^8 y^{12}}\right)^{1/4}$

$= \dfrac{3^{\cancel{4}\times\frac{1}{\cancel{4}}}}{x^{\cancel{8}\times\frac{1}{\cancel{4}}}\cdot y^{\cancel{12}\times\frac{1}{\cancel{4}}}} = \dfrac{3}{x^2 y^3}$

ii) $\sqrt{25x^{10n}y^{8m}}$

$= (25x^{10n}\cdot y^{8m})^{1/2}$

$= (5^2\cdot x^{10n}\cdot y^{8m})^{1/2}$

$= 5^{\cancel{2}\times\frac{1}{\cancel{2}}}\, x^{\cancel{10}n\times\frac{1}{\cancel{2}}}\cdot y^{8m\times\frac{1}{\cancel{2}}}$

$= 5\, x^{5n}\, y^{4m}$ Ans

iii) $\left[\dfrac{x^3\ y^4\ z^5}{x^{-2}\ y^{1}\ z^{-5}}\right]^{1/5}$

$= [x^{3+2}\cdot y^{4+1}\cdot z^{5+5})^{1/5}$

$= (x^5\ y^5\ z^{10})^{1/5}$

$= x^{\cancel{5}\times\frac{1}{\cancel{5}}}\cdot y^{\cancel{5}\times\frac{1}{\cancel{5}}}\cdot z^{\cancel{10}\times\frac{1}{\cancel{5}}}$

$= x\cdot y\cdot z^2$ Ans

Q7 Simplify Imp

$$\sqrt[3]{\frac{a^l}{a^m}} \times \sqrt[3]{\frac{a^m}{a^n}} \times \sqrt[3]{\frac{a^n}{a^l}}$$

$$= \frac{a^{l \times \frac{1}{3}}}{a^{m \times \frac{1}{3}}} \times \frac{a^{m \times \frac{1}{3}}}{a^{n \times \frac{1}{3}}} \times \frac{a^{n \times \frac{1}{3}}}{a^{l \times \frac{1}{3}}}$$

$$= \frac{a^{l/3}}{a^{m/3}} \times \frac{a^{m/3}}{a^{n/3}} \times \frac{a^{n/3}}{a^{l/3}}$$

$$= 1 \text{ Ans}$$

Q8 Simplify Imp

$$\left(\frac{a^p}{a^r}\right)^{p+r} \cdot \left(\frac{a^q}{a^r}\right)^{q+r} \div 5(a^p \cdot a^r)^{p-r}, a \neq 0$$

$$= (a^{p-q})^{p+q} \cdot (a^{q-r})^{q+r} \div 5(a^{p+r})^{p-r}$$

$$= a^{(p-q)(p+q)} \cdot a^{(q-r)(q+r)} \div 5a^{(p+r)(p-r)}$$

$$= a^{p^2-q^2} \cdot a^{q^2-r^2} \div 5a^{p^2-r^2}$$

$$= a^{p^2-\cancel{q^2}+\cancel{q^2}-r^2} \div 5a^{p^2-r^2}$$

$$= \frac{a^{p^2-r^2}}{5a^{p^2-r^2}}$$

$$= \frac{a^{\cancel{p^2}-\cancel{r^2}-\cancel{p^2}+\cancel{r^2}}}{5}$$

$$= \frac{a^0}{5} = \frac{1}{5} \text{ Ans}$$