Chapter # 12.

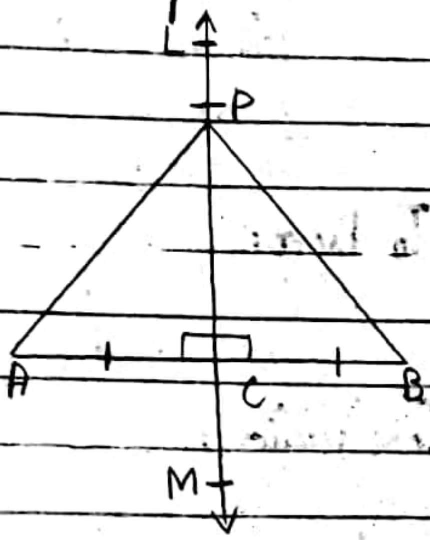
"Line Bisector and Angle Bisectors"

→ **Theorem 12.1.1**

Any point on the right bisector of a line segment is equidistant from its end point.

Given:

A line LM intersects the line segment AB at the point C such that $\overleftrightarrow{LM} \perp \overline{AB}$ and $\overline{AC}$ and $\overline{AC} \cong \overline{BC}$. P is a point on $\overleftrightarrow{LM}$.



To prove:

$\overline{PA} \cong \overline{PB}$

Construction:

Join P to the point A and B.

Proof:

Statements	Reasons
I $\Delta ACP \longleftrightarrow \Delta BCP$	
$\overline{AC} \cong \overline{BC}$	given
$\angle ACP \cong \angle BCP$	given $\overline{PC} \perp \overline{AB}$, so that each $\angle$ at $C = 90^\circ$
$\overline{PC} \cong \overline{BC}$	common
$\therefore \Delta ACP \cong \Delta BCP$	S.A.S. postulate
Hence $\overline{PA} \cong \overline{PB}$	(corresponding sides of congruent triangle).

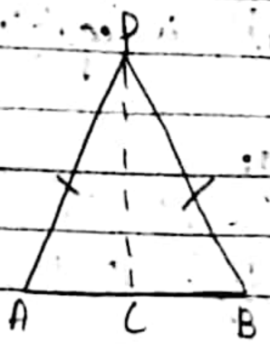
Scanned with CamScanner

Theorem 12.1.2:

Any point equidistant from the end points of a line segment is on the right bisector of it.

Given:

AB is a line segment.
Point P is such that
$\overline{PA} \cong \overline{PB}$.



To Prove:

The point P is on the right bisector of $\overline{AB}$.

Construction:

Join P to C, the mid-point of $\overline{AB}$.

Proof

Statements	Reason
In $\Delta ACP \longleftrightarrow \Delta BCP$	
$\overline{PA} \cong \overline{PB}$	given
$\overline{PC} \cong \overline{PC}$	common
$\overline{AC} \cong \overline{BC}$	construction
$\Delta ACP \cong \Delta BCP$	S.S.S ≅ S.S.S
$\angle ACP \cong \angle BCP$ ....(i)	(corresponding angles of congruent triangle).
But $m\angle ACP + m\angle BCP = 180^\circ$ ..(ii)	Suplementry angles
$\therefore m\angle ACP = m\angle BCP = 90^\circ$	from (i) and (ii)
i.e., $\overline{PC} \perp \overline{AB}$ ..... (iii)	$m\angle ACP = 90^\circ$ (proved)
Also $\overline{CA} \cong \overline{CB}$ ..... (iv)	construction
$\therefore$ $\overline{PC}$ is a right bisector of $\overline{AB}$.	

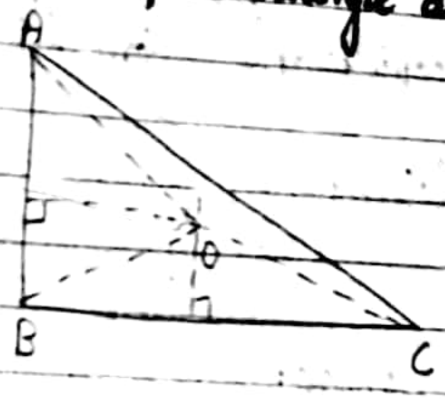
i.e., the point P is on the right bisector of $\overline{AB}$.	

→ **Theorem 12.1.3**

The right bisectors of the sides of a triangle are concurrent.

Given:

ΔABC



To prove

The right bisectors of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are concurrent.

Construction:

Draw the right bisectors of $\overline{AB}$ and $\overline{BC}$ which meet each other at the point O. Join O to A, B and C.

Proof:

Statements	Reasons
$\overline{OA} \cong \overline{OB}$ ....(i)	(Each point on right bisector of a segment is equidistant from its end points)
$\overline{OB} \cong \overline{OC}$ ....(ii)	as in (i)
$\overline{OA} \cong \overline{OC}$ ....(iii)	from (i) and (2)
∴ Point O is on the right bisector of $\overline{CA}$. ....(iv)	(O is equidistant from A and C)
But point O is on the right bisector of $\overline{AB}$ and $\overline{BC}$	Construction

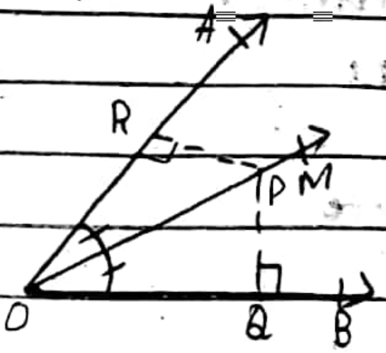
Scanned with CamScanner

Scanned with CamScanner

Hence the right bisectors of the three sides of a triangles are concurrent at O.	{From (iv) and (v)}

Theorem 12.1.4:

Any point on the bisector of an angle is equidistant from its arms.



Given:

A point P is on $\overrightarrow{OM}$, the bisector of $\angle AOB$.

To prove:

$\overline{PQ} \cong \overline{PR}$ i.e., P is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$

Construction:

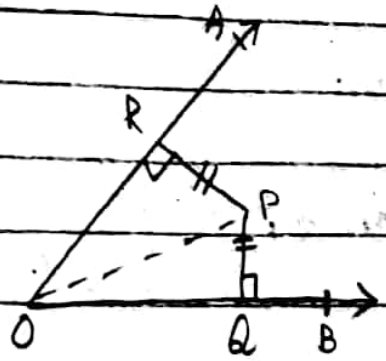
Draw $\overline{PR} \perp \overrightarrow{OA}$ and $\overline{PQ} \perp \overrightarrow{OB}$

Proof

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	common
$\angle PQO \cong \angle PRO$	construction
$\angle POQ \cong \angle POR$	given
$\therefore \Delta POQ \cong \angle POR$	S.A.A $\cong$ S.A.A
Hence $\overline{PQ} \cong \overline{PR}$	(corresponding sides of congruent triangle)

Theorem 12.1.5

Any point inside an angle, equidistant from its arm, is on the bisector of it.



Given:

Any point P lies inside $\angle AOB$ such that $\overline{PQ} \cong \overline{PR}$, where $\overline{PQ} \perp \overrightarrow{OB}$ and $\overline{PR} \perp \overrightarrow{OA}$.

To prove:

Point P is on the bisector of $\angle AOB$.

Construction:

Join P to O

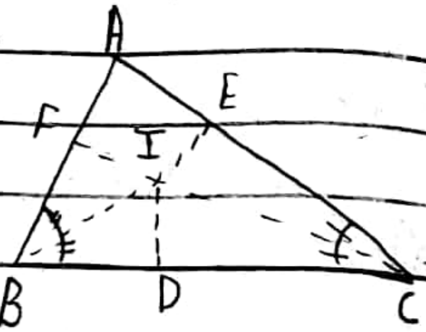
Proof:

Statements	Reasons
In $\Delta POQ \longleftrightarrow \Delta POR$	
$\angle PQO \cong \angle PRO$	given right angles
$\overline{PO} \cong \overline{PO}$	common
$\overline{PQ} \cong \overline{PR}$	given
$\therefore \Delta POQ \cong \Delta POR$	H.S $\cong$ H.S
Hence $\angle POQ \cong \angle POR$	(corresponding angles of congruent triangle).
i.e., P is on the bisector of $\angle AOB$.	

Scanned with CamScanner

Theorem 12.1.6

The bisectors of the angles of a triangle of are concurrent.



Given:

ΔABC

To prove:

The bisectors of $\angle A, \angle B$ and $\angle C$ are concurrent.

Construction:

Draw the bisectors of $\angle B$ and $\angle C$ which intersects at point I. From I, draw $\overline{IF} \perp \overline{AB}$, $\overline{ID} \perp \overline{BC}$ and $\overline{IE} \perp \overline{CA}$.

Prove

Statements	Reasons
$\overline{ID} \cong \overline{IE}$	(Any point on bisector of an angle is equidistant from its arms)
Similarly, $\overline{ID} \cong \overline{IF}$	
$\overline{IE} \cong \overline{IF}$	Each $\cong \overline{ID}$, proved
So, the point I is on the bisector of $\angle A$ ...(i)	
Also the point I is on the bisector of $\angle ABC$ and $\angle BCA$ ...(ii)	Construction
Thus the bisectors of $\angle A, \angle B$ and $\angle C$ are concurrent at I	{from (i) and (ii)}

Scanned with CamScanner