Exercise # 4.3

1- Simplify

i- $2^7 \div 2^2$

sol:- $= 2^7 \div 2^2$

$= 2^{7-2} \quad \because a^m \div a^n = a^{m-n}$

$= 2^5$

ii- $(-9)^{11} \div (-9)^8$

sol $(-9)^{11} \div (-9)^8$

$= (-9)^{11-8} \quad \because a^m \div a^n = a^{m-n}$

$= (-9)^3$

iii- $(3)^4 \div (5)^4$

sol $(3)^4 \div (5)^4$

$= \left(\frac{3}{5}\right)^4 \quad \because a^n \div b^n = \left(\frac{a}{b}\right)^n$

Do Q1 (iv), (v) & (vi) by same method.

(vii) $\left(\frac{3}{4}\right)^7 \div \left(\frac{3}{4}\right)^2$

Sol $\left(\frac{3}{4}\right)^7 \div \left(\frac{3}{4}\right)^2$

$= \left(\frac{3}{4}\right)^{7-2}$ $\because a^m \div a^n = a^{m-n}$

$= \left(\frac{3}{4}\right)^5$

(viii) $\left(\frac{1}{6}\right)^{15} \div \left(\frac{1}{6}\right)^{11}$

Sol $\left(\frac{1}{6}\right)^{15} \div \left(\frac{1}{6}\right)^{11}$

$= \left(\frac{1}{6}\right)^{15-11}$ $\because a^m \div a^n = a^{m-n}$

$= \left(\frac{1}{6}\right)^4$

(ix)

Sol

=

2-

(i)

ii- $(-4)^3 \div (5)^3 = \left(\frac{-4}{5}\right)^3$

L.H.S ⟶

$= (-4)^3 \div (5)^3 = \frac{(-4)^3}{(5)^3}$

$= \frac{-4 \times -4 \times -4}{5 \times 5 \times 5}$

$= \left(\frac{-4}{5}\right)^1 \times \left(\frac{-4}{5}\right)^1 \times \left(\frac{-4}{5}\right)^1 = \left(\frac{-4}{5}\right)^{1+1+1}$

$= \left(\frac{-4}{5}\right)^3 =$ R.H.S

(iii) $3^8 \div 3$

L.H.S ⟶

$= 3^8 \div 3$

$= \frac{3^8}{3} = \frac{\cancel{3} \times 3^1 \times 3^1 \times 3^1 \times 3^1 \times 3^1 \times 3^1 \times 3^1}{\cancel{3}}$

$= 3^{1+1+1+1+1+1+1}$

$= 3^7$

(ix) $(2)^5 \div (3)^5$

Sol $(2)^5 \div (3)^5$

$= \left(\frac{2}{3}\right)^5 \quad \because a^n \div b^n = \left(\frac{a}{b}\right)^n$

Do Q1(x), (xi) & (xii) by same method

2- Prove that

(i) $2^4 \div 7^4 = \left(\frac{2}{7}\right)^4$

L.H.S $\longrightarrow$

$= 2^4 \div 7^4$

$= \frac{2^4}{7^4} = \frac{2 \times 2 \times 2 \times 2}{7 \times 7 \times 7 \times 7}$

$= \left(\frac{2}{7}\right) \times \left(\frac{2}{7}\right) \times \left(\frac{2}{7}\right) \times \left(\frac{2}{7}\right)$

$= \left(\frac{2}{7}\right)^{1+1+1+1} = \left(\frac{2}{7}\right)^4 = R.H.S$

(v) $\left(\frac{-21}{22}\right)^7 \div \left(\frac{-21}{22}\right)^3 = \left(\frac{-21}{22}\right)^4$

L.H.S $\longrightarrow$

$= \left(\frac{-21}{22}\right)^7 \div \left(\frac{-21}{22}\right)^3$

$= \frac{\left(\frac{-21}{22}\right)^7}{\left(\frac{-21}{22}\right)^3}$

$= \frac{\left(\frac{-21}{22}\right)\times\left(\frac{-21}{22}\right)\times\left(\frac{-21}{22}\right)\times\left(\frac{-21}{22}\right)\times\left(\frac{-21}{22}\right)\times\left(\frac{-21}{22}\right)\times\left(\frac{-21}{22}\right)}{\left(\frac{-21}{22}\right)\times\left(\frac{-21}{22}\right)\left(\frac{-21}{22}\right)}$

$= \left(\frac{-21}{22}\right)\times\left(\frac{-21}{22}\right)\times\left(\frac{-21}{22}\right)\times\left(\frac{-21}{22}\right)$

$= \left(\frac{-21}{22}\right)^{1+1+1+1} = \left(\frac{-21}{22}\right)^4 = \text{R.H.S}$

Q2 (iv) & (vi) do by same method.

Ex # 4.4

1 Express the following as single exponents

(i) $(2^3)^5$

Sol = $(2^3)^5$

$= 2^{3\times5}$ $\because (a^m)^n = a^{mn}$

$= 2^{15}$

(ii) $(10^2)^2$

Sol = $(10^2)^2$

$= 10^{2\times2}$ $\because (a^m)^n = a^{mn}$

$= 10^4$

(iii) $[(-3)^4]^5$

$= [(-3)^4]^5$

$= (-3)^{4\times5}$ $\because (a^m)^n = a^{mn}$

$= (-3)^{20}$

(iv) $(P^2)^3$

$= (P^2)^3$

$= (P)^{2\times3}$ $\because (a^m)^n = a^{mn}$

$= P^6$

(vii) $\left[\left(-\frac{1}{3}\right)^3\right]^3$

$= \left[\left(-\frac{1}{3}\right)^3\right]^3$

$= \left(\frac{-1}{3}\right)^{3\times3}$ $\because (a^m)^n = a^{mn}$

$= \left(-\frac{1}{3}\right)^9$

Do Q 1 (v), (viii) & (ix) by same method.

Q2 Change the following negative exponents into positive exponents

(i) $(12)^{-3}$

$= \left(\frac{1}{12}\right)^{3}$

(ii) $(-a)^{-2}$

$= \left(\frac{1}{-a}\right)^{2}$

(iii) $(100)^{-5}$

$= \left(\frac{1}{100}\right)^{5}$

(iv) $\left(\frac{2}{3}\right)^{-4}$

$= \left(\frac{3}{2}\right)^{4}$

(v) $\left(\frac{x}{y}\right)^{-b}$

$= \left(\frac{y}{x}\right)^{b}$

(vi) $\left(\frac{-1}{10}\right)^{9}$

Q3 Evaluate the following expression

(i) $(1^2)^3 \times (2^3)^2$

$= (1^2)^3 \times (2^3)^2$

$= (1)^{2\times3} \times (2)^{3\times2}$ $= (a^m)^n = a^{mn}$

$= (1)^6 \times (2)^6$

$= (1\times2)^6 = 2^6 = 64$

(ii) $[(-3^7)]^0 \times [(-3)^2]^2$

$= [(-3)^7]^0 \times [(-3)^2]^2$

$= (3)^{7\times0} \times (-3)^{2\times2}$

$= (3)^0 \times (-3)^4$

$= 1 \times 81$

$= 81$

(iii) $\left[\left(\frac{-3}{4}\right)^0\right]^3 \times \left[\left(\frac{-3}{4}\right)^2\right]^2$

$= \left(\frac{-3}{4}\right)^{0\times3} \times \left(\frac{-3}{4}\right)^{2\times2}$

$= \left(\frac{-3}{4}\right)^0 \times \left(\frac{-3}{4}\right)^4$

$= 1 \times \frac{81}{256} = 81/256$

(iv) $\left[\frac{2^3}{2^6 \div 2^3}\right]$

$= \left[\frac{2^3}{2^6 \div 2^3}\right] = \left[\frac{2^3}{2^{6-3}}\right] \because a^m \div a^n = a^{m-n}$

$= \frac{2^3}{2^3} = 2^{3-3} = 2^0$

$= 1$

(vi) $\frac{\left(\frac{-2}{9}\right)^5 \times \left(\frac{-2}{9}\right)^{-5}}{\left(\frac{3}{2}\right)^4 \times \left(\frac{3}{2}\right)^{-4}}$

$= \frac{\left(\frac{-2}{5}\right)^{5+(-5)}}{\left(\frac{3}{2}\right)^{4+(-4)}}$ $\because a^m \times a^n = a^{m+n}$

$= \frac{\left(\frac{-2}{5}\right)^0}{\left(\frac{3}{2}\right)^0} = \frac{1}{1} = 1$

(vii) $\frac{\left(\frac{1}{3}\right)^{-3}}{\left(\frac{1}{3}\right)^{-4}} - \frac{\left(\frac{1}{3}\right)^{-5}}{\left(\frac{1}{3}\right)^{-6}}$

$= \left(\frac{1}{3}\right)^{-3-(-4)} - \left(\frac{1}{3}\right)^{-5-(-6)}$

$= \left(\frac{1}{3}\right)^{-3+4} - \left(\frac{1}{3}\right)^{-5+6}$

$= \frac{1}{3} - \frac{1}{3} = 0$

viii) $\frac{\left(\frac{2}{3}\right)^{-5} \times \left(\frac{2}{3}\right)^{4}}{\left(\frac{2}{3}\right)^{-4} \div \left(\frac{2}{3}\right)^{-4}}$

$= \frac{\left(\frac{2}{3}\right)^{-5+4}}{\left(\frac{2}{3}\right)^{-4-(-4)}}$ $\because a^m \times a^n = a^{m+n}$ $\because a^m \div a^n = a^{m-n}$

$= \frac{\left(\frac{2}{3}\right)^{-1}}{\left(\frac{2}{3}\right)^{-4+4}} = \frac{\left(\frac{2}{3}\right)^{-1}}{\left(\frac{2}{3}\right)^{0}}$

$= \frac{\left(\frac{3}{2}\right)^{1}}{1} = \frac{3}{2}$

(ix) $\left(\frac{2}{3}\right)^{3} \times \left(\frac{2}{3}\right)^{0} \times \left(\frac{2}{3}\right)^{-3}$

$= \left(\frac{2}{3}\right)^{3+0+(-3)}$ $= a^m \times a^n = a^{m+n}$

$= \left(\frac{2}{3}\right)^{3+0-3} = \left(\frac{2}{3}\right)^{0} = 1$

(x) $\left(\frac{-1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{4}\right)^{-4}$

$= (2)^2 + (3)^3 + (4)^4 \quad \because a^{-n} = \left(\frac{1}{a}\right)^n$

$= 4 + 27 + 256$

$= 287$